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FAILURE OF PLAIN CONCRETE UNDER COMBINED STRESSES

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FAILURE OF PLAIN CONCRETE UNDER COMBINED STRESSES

Boris Bresler,[1] A.M. ASCE and Karl S. Pister,[2] J.M. ASCE

ABSTRACT

The initial phase of an investigation of the failure of plain concrete under combined stresses is presented. Previous investigations of the failure of plain concrete reveal that the correlation of various criteria with test data has been either inadequate or inconclusive. To obtain further evidence on failure of plain concrete, hollow cylinders were tested under varying combinations of torsion and compression. Data obtained from these tests indicated a correlation between normal and shearing octahedral stresses at failure. Within the limits of available data the octahedral stress criterion was generalized to include the effect of the third stress invariant, and in this form the proposed criterion was shown to be in good agreement with the previously reported data available to the authors.

INTRODUCTION

Knowledge of criteria of failure for concrete subjected to combined stress is of considerable practical importance. In considering the mechanism of failure of reinforced or prestressed concrete, it has often been stated that presence of reinforcement or prestress alters the nature of the material and therefore changes the criteria of failure. Indeed, bond between concrete and reinforcement and deformation in the reinforcement play a very important role in the failure mechanism. It seems reasonable, however, that the conditions producing local failure in concrete are essentially the same for both plain and reinforced concrete. Therefore, knowledge of the conditions producing local failure in plain concrete would lead to a better understanding of the failure mechanism in reinforced or prestressed structural elements.

Formulation of failure criteria for concrete has received considerable attention in the past. In engineering practice the most commonly proposed theories are those of maximum stress, internal friction, and maximum strain. The maximum stress theory is unsatisfactory as it fails to agree with experimental data for biaxial and triaxial states of stress.(1, 2, 3, 4).[3]

Mohr's generalization of the internal friction theory has been proposed for concrete.(4, 5) Correlation of this theory with experimental data is not entirely satisfactory, at least in part due to the approximation involved in the assumption that the failure criterion is independent of the intermediate principal stress. Torre(6) demonstrated that for a given material there are as many Mohr envelopes as there are values of a parameter α, defined as $\alpha = (\sigma_2 - \sigma_1)/(\sigma_1 - \sigma_3)$, where $\sigma_1 > \sigma_2 > \sigma_3$ are principal stresses. It follows

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3. Numbers in parentheses indicate references listed at the end of the paper.

that construction of a Mohr envelope from test data in which is not constant cannot be justified.

The maximum strain theory of failure states that failure occurs when either maximum tensile or maximum compressive strain reaches a limiting value. For linearly elastic and isotropic materials this criterion leads to a simple mathematical formulation. For inelastic and anisotropic materials, such as concrete, this theory is not readily applicable. Approximate criteria of failure in terms of maximum concrete strain have been proposed for flexure in reinforced concrete members.(7) Comparison of these criteria with experimental data(7) shows considerable scatter, and the general use of these approximations cannot be justified.

Cowan(8) proposed criteria based on two distinct types of failure of plain concrete. One criterion, associated with a cleavage fracture, states that failure is governed by maximum tensile stress or strain. The other criterion, associated with crushing strength, states that failure is governed essentially by the Coulomb internal friction law. Cowan's published experimental data(9) are limited to reinforced concrete elements subjected to bending and torsion and are insufficient for evaluating the validity of his hypothesis of failure for plain concrete.

In an effort to account for the heterogeneous nature of concrete, Brandtzaeg(10) developed a theory based on an idealized (model) structure of the material. This theory is limited to cylindrical states of stress, i.e., two of the three principal stresses are equal, and considers the possibility of distinct shear or cleavage failure. Application of this theory defines a stress-strain relationship and a "critical" stress corresponding to ultimate strength of the material. A series of biaxial and triaxial tests of concrete conducted at the University of Illinois by Richart, Brandtzaeg, and Brown(3) indicated reasonable agreement with Brandtzaeg's theoretical stress-strain relationship for stresses below the "critical" value, but showed large discrepancies between the "critical" stress and the ultimate strength.

Freudenthal(1) discussed the problem of formulating failure criteria for concrete and proposed a linear equation in terms of effective stress σ_e and mean stress p as follows:

$$\sigma_e = c_1 p + c_2 \tag{1}$$

where

$$\sigma_e = \left\{ 1/2 \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \right\}^{1/2} \tag{2}$$

$$p = 1/3 \, (\sigma_1 + \sigma_2 + \sigma_3) \tag{3}$$

and c_1 and c_2 are constants of the material. To verify Eq. (1) for biaxial states of stress Freudenthal carried out a series of compression tests on notched cylinders. As pointed out by Freudenthal it is doubtful that the notch stresses computed by elastic theory actually develop at failure; therefore experimental evidence presented cannot be considered conclusive.

In addition to the theories discussed above, various energy criteria for failure have been proposed for brittle materials.(11, 12) Griffith(11) proposed

a fracture theory which considers the effect on the total energy of the formation of macroscopic cracks within the material. For two cases of plane stress, assuming the cracks to be elliptic, Griffith obtained a theoretical tensile strength of fracture. It may be noted that for materials that exhibit non-linear elasticity the Griffith theory is no longer interpretable in terms of elastic strain energy. Likewise, for materials subjected to three-dimensional states of stress, the solutions for plane stress become approximations. The philosophy expressed by the Griffith theory seems to have found utility in the studies of failure of cast iron by Fisher(13) and Grassi and Cornet.(14) A preliminary investigation by Wollak(2) revealed that for concrete the agreement of the available experimental evidence with Griffith's theory is not entirely satisfactory.

Existing failure criteria for plain concrete are neither adequately substantiated nor disproved. Most of the experimental data on strength of plain concrete has been obtained for specimens subjected to uniaxial, biaxial, or triaxial compression, uniaxial tension, and flexure. To the best of the authors' knowledge data on failure of concrete subjected to shear-compression states of stress have not been obtained previously. To obtain experimental evidence for these states of stress twenty-four hollow cylinders were tested under varying combinations of torsion and compression. The objectives of these tests were to obtain further verification of the various existing theories of failure, to correlate the new data with previously reported investigations, and to assist in formulating a more general criterion of failure. Data obtained from these tests indicated a linear relation between normal and shearing octahedral stresses at failure. Previously reported data for biaxial states of stress agrees reasonably well with the established linear relationship, but data for triaxial states of stress deviate from it. This deviation is shown to be related to the third invariant of the stress tensor, $I_3 = \sigma_1 \sigma_2 \sigma_3$. A generalized form of a failure criterion based on octahedral stresses and the third stress invariant is proposed, and possible physical interpretations are discussed.

Test Specimens and Procedure

Hollow concrete cylinders, 9 in. O.D., 6 in. I.D., and 30 in. long were made using mix proportions of 1:2.9:3.1 by weight, and ratio of water to cement of 0.6 by weight. One-half inch maximum aggregate size was used and the slump averaged about 2 inches. All specimens were cured at 100 per cent humidity and 70° F. and tested at the age of 14 days.

Four 3 x 6 in. control cylinders were made for each specimen and tested in compression at the age of 14 days. The average compressive strength of these cylinders was taken as f'_c for the particular specimen.

Compression tests of hollow cylinders were made in a 200-kip Baldwin-Southwark hydraulic testing machine using a load rate of 10 kpm. Pure torsion tests were made using special loading and anchor arms which were attached to the specimen by steel bands. The load was applied by means of an 8-in. screw jack and the torque was computed as the product: (jack load) x (lever arm of 45 in.). The load on the jack was measured by means of a calibrated proving ring with SR-4 strain gages.

Combined loads were obtained by using the torque apparatus and a 300 K Riehle screw gear testing machine, as shown in Fig. 1.

In combined load tests a predetermined amount of compression was applied first, and then the torque was applied to produce failure; the compression load

was kept very nearly constant during the test. The specimen was carefully centered in the testing machine, and the load was applied through a spherical bearing block. Special lubricated bearing plates were used to reduce friction between the top of the specimen and head of the machine. These bearing plates permitted measurement of friction torque during the test and, although this never exceeded 4 per cent of applied torque, the actual torque on the specimen was corrected for this friction.

Test Results

Typical failures obtained in this series of tests are shown in Fig. 2. Pure torsion specimens exhibited approximately 45 degree helical fractures due to diagonal tension, Fig. 2A. Under combined loadings the specimens failed by helical cracks, the helix angle of which varied from 19 to 34 degrees depending upon the amount of compression, Fig. 2B. Pure compression cylinders failed along irregular vertical, inclined, or circumferential cracks, Fig. 2C.

Stresses at failure were computed from the measured loads as follows: $\sigma = P/A$ and $\tau = TR/J$, where P is the compression load at failure, A is the area of test specimen, T is the corrected torque on the specimen at failure, R is the outside radius of hollow cylinder, and J is its polar moment of inertia. For groups of similar specimens average values of the compressive (σ) and shearing (τ) stresses at failure, the nominal compressive strength (σ_c)[4] and the helix angle of the major cracks are given in Table I.

Table I

Group	$-\sigma$ psi	τ psi	$-\sigma_c$ psi	θ^o
J K L	0	284	2710	49
N P R	400	430	2760	32
O Q BB	810	490	2790	26
CC DD EE FF	1050	500	2690	22
S T W GG	1280	620	2590	23
U X	1670	570	2640	--
V	2000	530	2530	--
Y	2180	570	2490	--
A C E F	2680	0	2680	--

Compressive stress σ was calculated assuming that the load was perfectly axial and that the stress distribution was uniform over the entire cross section. The shear stress τ was calculated on the basis of elastic theory which

4. Averaging results of test group ACEF the strength of hollow cylinders, σ_c was found to be equal to 86.6 per cent of f'_c of 3 x 6 in. companion control cylinders. Based on this relationship values of σ_c were computed for all specimens.

at failure is only an approximation. If concrete were perfectly plastic, the shear stress at failure would be approximately 85 per cent of the value given by elastic theory. Actual shear stress at failure is somewhere between these limits and for the purpose of this paper it is approximated by the elastic theory value.

Principal stress at failure σ_1 and σ_2 and dimensionless ratios (σ_1/σ_c) and $(\sigma_2/\ _c)$ were computed for each individual specimen. The dimensionless ratios are plotted in Fig. 3. With few exceptions the data follows a pattern which does not agree with either the maximum stress or Coulomb's internal friction theory.

It has been suggested[15] that yielding failure can be defined by the following criterion:

$$\tau_o = F\ (\sigma_o) \tag{4}$$

where σ_o and τ_o are the normal and shearing octahedral stresses. Octahedral stresses occur on the sides of an octahedral element formed by planes whose normals make equal angles with the principal stress axes, Fig. 4, and are defined by Eq. (5)

$$\begin{aligned} \sigma_o &= (1/3)\ (\sigma_1 + \sigma_2 + \sigma_3) \\ \tau_o &= (1/3)\ \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2} \end{aligned} \tag{5}$$

A dimensionless plot of Eq. (4) for the test data is shown in Fig. 5. Fitting a straight line to the data by connecting the points obtained by averaging results of pure torsional tests and pure compressive tests yields the equation

$$(-\tau_o/\sigma_c) = 1.15\ (\sigma_o/\sigma_c) + 0.087 \tag{6}$$

To further test the validity of Eq. (6) results of a number of tests in which concrete was subjected to combined states of stress were considered. Of particular interest are biaxial tests[1] and triaxial tests.[3, 4] The principal characteristics of the specimens and the test conditions are summarized in Table II; other data relating to specimens and test procedures can be found in the references cited.

It is apparent from Table II that the data represent concretes of different ages, mixes, strengths, and moisture conditions at time of test. The effect of these varying characteristics on the failure criteria cannot be readily defined. In this analysis the nominal compressive strength σ_c is used to account for the difference in the nature of the materials. As previously indicated, for hollow cylinders σ_c was determined as 86.6 per cent of the strength of 3 by 6 in. control cylinders, for Richart's and Balmer's specimens σ_c was taken as the compressive strength of corresponding control cylinders, and for Freudenthal's series of tests σ_c was taken as the compressive strength of unnotched control cylinders.

Octrahedral stresses σ_o and τ_o at failure and dimensionless ratios of (σ_o/σ_c) and (τ_o/σ_c) were computed for individual specimens, and average stresses and ratios were computed for groups of identical specimens. The

Table II

Source		Richart et al. (3)	Balmer (4)	Freudenthal (1)
Type of Loading		Triaxial Series 3A *	Triaxial	Biaxial Series **
Specimen Type and Size		Cylinder 8 in. long 4 in. diam.	Cylinder 12 in. long 6 in. diam.	Notched cylinder 10 in. long 4 in. diam.
No. of Tests		64	50	23
Mix	Proportion	1:2.15:2.5 1:1:2 1:3:5	1:2.9:4.5	--
Mix	Max. Aggr.	3/4 in.	1-1/2 in.	--
Mix	W/C	0.88 0.64 1.25	0.58	--
Curing Cond.	Humid %	moist	100	100 and 70
Curing Cond.	Temp. °F	--	70	70
Duration, days		27	28	14
Age, days		28	35 and 97	28
Test. Cond.		air dry	oven dry	--
σ_c range, psi		1050-3660	3570-3950	2960-9360

* Series 2 and 3-B are omitted. Results uncertain due to mechanical difficulties encountered in the tests.

** Tests with sustained load and fast loading rates are omitted.

dimensionless ratios are plotted in Fig. 6. Considering the heterogeneity of concrete and the variations in the type and in the conditions of tests, the scatter of the data is not significant. Furthermore, it appears that the data in the biaxial stress range can be adequately represented by a straight line, but that data corresponding to triaxial states of stress deviate appreciably from the assumed linear relationship. Since the third invariant I_3 [5] is identically zero for biaxial states of stress and non zero for triaxial states of stress, it appears plausible that the deviation of triaxial data from the biaxial linear relationship may be related to the third stress invariant.

In a general case, any invariant symmetric function depending upon state of stress (e.g., a criterion of failure) is completely defined in terms of the three principal stress invariants. Therefore, a generalized failure criterion can be written as $F(I_1, I_2, I_3 = 0)$. This concept has been applied to the development of yield criteria for plastic materials by Prager and Hodge.[16]

5. The principal invariants are defined as $I_1 = \sigma_1 + \sigma_2 + \sigma_3$, $I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$, $I_3 = \sigma_1\sigma_2\sigma_3$.

Octahedral stresses defined by Eq. (5) may also be expressed in terms of principal stress invariants I_1 and I_2 [5] as

$$\sigma_o = (1/3)\, I_1 \quad \text{and} \quad \tau_o = (1/3)\left[2(I_1 - 3I_2)\right]^{1/2} \tag{5'}$$

and the failure criterion defined by Eq. (4) can be written as $F(I_1, I_2) = 0$. Therefore, a generalized form of the failure criterion $F(I_1, I_2, I_3) = 0$ may be written in a form similar to Eq. (4) as follows:

$$\tau_o = F_1'\,(\sigma_o) + F_2'\,(I_3) \tag{7}$$

where F_1' and F_2' are functions characteristic of the material. Rewriting Eq. (7) in dimensionless form gives

$$(\tau_o / \sigma_c) = F_1\,(\sigma_o/\sigma_c) + F_2\,(I_3/\sigma_c^3) \tag{8}$$

For biaxial states of stress $I_3 = 0$, and Eq. (8) gives the following

$$(\tau_o/\sigma_c) = F_1\,(\sigma_o/\sigma_c) + F_2\,(0) \tag{9}$$

It is seen from Fig. 6 that for states of stress for which I_3 is not zero the data deviate appreciably from the straight line defined by Eq. (6). This deviation, denoted by δ in Fig. 6, can be computed from test data. In terms of the proposed failure criterion δ represents the difference between Eqs. (9) and (8):

$$\delta = F_2\,(0) - F_2\,(I_3/\sigma_c^3) \tag{10}$$

Correlation between computed values of δ and corresponding values of (I_3/σ_c^3) would support the generalized failure criterion proposed in Eqs. (7) and (8). A logarithmic plot of δ vs. (I_3/σ_c^3), Fig. 7, indicates a very satisfactory correlation between these parameters, providing some evidence for the validity of the proposed hypotheses.

DISCUSSION

In this paper a general form of failure criterion for concrete is proposed by Eq. (7) stating that failure occurs when the octahedral shear stress τ_o reaches a limiting value which is a function of the octahedral normal stress σ_o and the third stress invariant I_3. The validity of this criterion is limited to the following: a single loading to failure is considered, the duration of loading is relatively short, the octahedral normal stress σ_o is compressive or zero, and the stress distribution is approximately uniform. Evaluation of the effects of repeated cycles of loading, sustained loading, octahedral tensile stress, and non-uniform stress distribution requires additional experimental evidence.

There is no direct consideration of the effect of composition and structure

of the material, rate and manner of loading, type and size of specimen, stress history, and volume changes depending upon stress, humidity, and temperature. To obtain a criterion which would take into account these various factors requires knowledge of the physical and chemical structure of concrete beyond that presently available. While basic research to determine the effect of structure on strength of concrete is urgently needed, for the time being a phenomenological analysis of failure appears to be the most fruitful approach.

An interesting physical interpretation of the octahedral shearing stress has recently been pointed out by Novozhilov.(17) [6] Consider at a given point the shearing stress τ on a plane defined by a normal whose direction cosines referred to principal axes are 1, m, and n. In terms of the principal stresses and the direction cosines it is known that

$$\tau^2 = \sigma_1^2 l^2 + \sigma_2^2 m^2 + \sigma_3^2 n^2 - (\sigma_1 l^2 + \sigma_2 m^2 + \sigma_3 n^2)^2 \quad (11)$$

The root mean square shearing stress τ_a on a closed surface S containing the point is defined as follows:

$$\tau_a = \lim_{S \to 0} \left[(1/S) \int_S \tau^2 \, dS \right]^{1/2} \quad (12)$$

In general the value of τ_a will depend upon the surface S over which the integration is performed. However, by choosing S to be the surface of a sphere of radius r, all possible directions are weighted equally, and an "average" shearing stress at the point is obtained. Carrying out the integration leads to

$$\tau_a^2 = (1/15) \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] = (9/15) \tau_o^2 \quad (13)$$

Eq. (13) indicates that the "average" shearing stress τ_a differs from octahedral shearing stress τ_o only by a numerical constant. Using this formulation and noting that octahedral normal stress σ_o is the "average" normal stress, the criterion of failure given by Eq. (6) can be written as follows:

$$(-\tau_a/\sigma_c) = 1.48 \, (\sigma_o/\sigma_c) + 0.112 \quad (14)$$

It is evident that for the biaxial states of stress considered failure depends on the magnitudes of the average shearing and normal stresses.

The failure criteria proposed above are entirely phenomenological. However, some of the parameters used in these criteria may be associated with a physical interpretation of failure mechanism. Consider four types of possible failure: slip between grains,[7] slip within a grain, cleavage between grains, and cleavage within a grain. It seems plausible that such failures may be associated with the average shearing and normal stresses as defined

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6. The authors are indebted to Prof. G. A. Zizicas for bringing this paper to their attention.
7. For the purpose of this discussion grains are defined as any small particle of material.

by Eq. (14). In case of triaxial compression deviation of the experimental data from the simple relationship of Eq. (14) may be associated with the limited effectiveness of grain interlocking in increasing the resistance to intergranular slip.

Other interpretations of failure criteria based on considerations of strain and/or energy are possible. Verification of such formulations may provide some useful information regarding the mechanism of failure in concrete. It seems, however, that until the intragranular and intergranular behavior of concrete under load is better understood a phenomenological criterion of failure, such as proposed here, is the most satisfactory.

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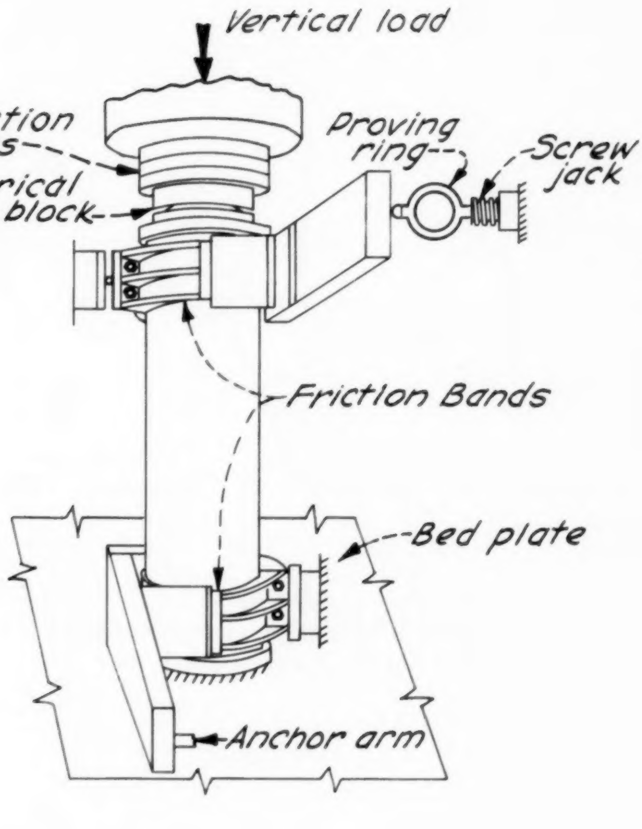
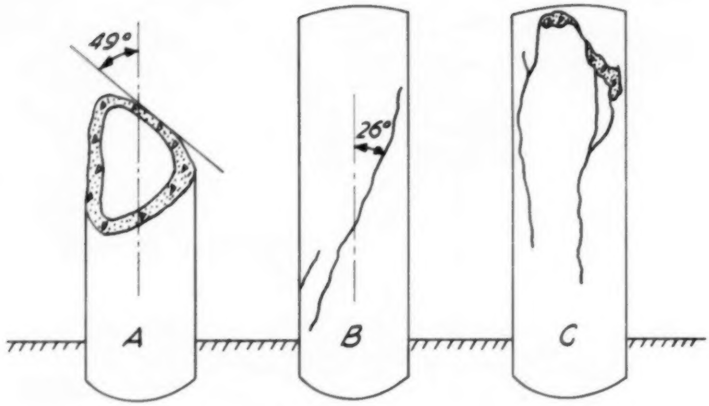

Vertical load
Low friction plates
Proving ring
Screw jack
Spherical bearing block
Friction Bands
Bed plate
Anchor arm

FIG. 1



A--Pure torsion
B--Torsion + compression
C--Pure compression

FIG. 2

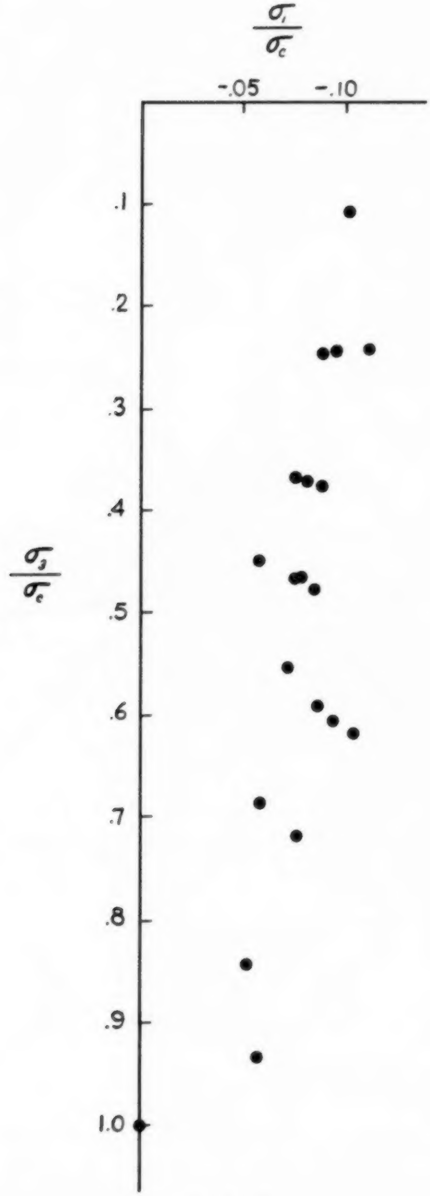


FIG. 3

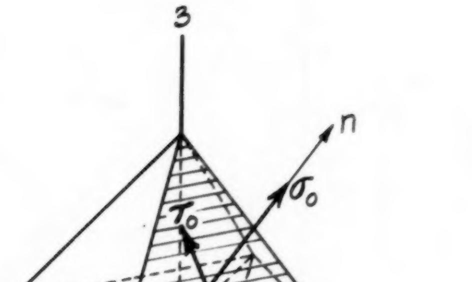


FIG. 4

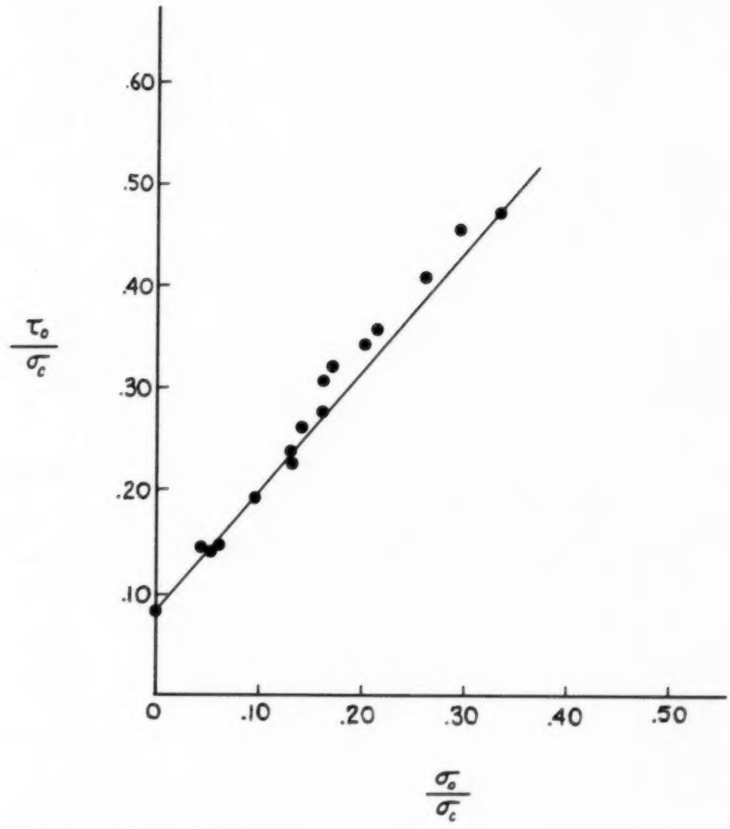
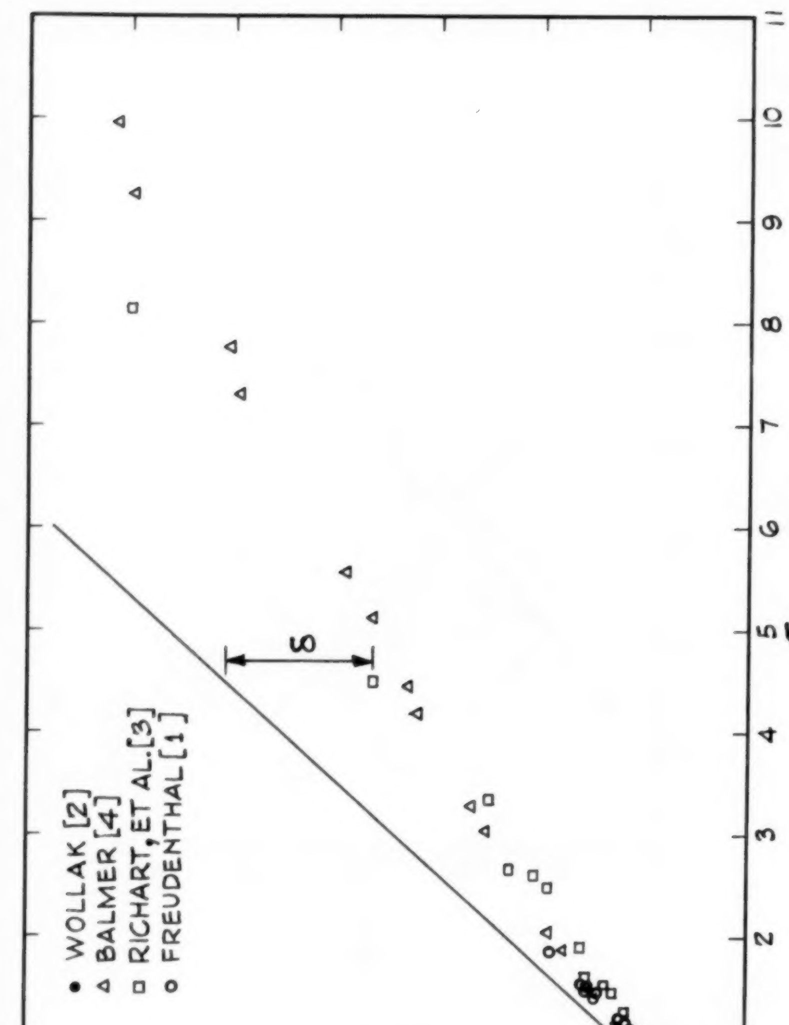

.60
.50
.40
.30
.20
.10
0
.10
.20
.30
.40
.50
$\frac{\tau_o}{\sigma_c}$
$\frac{\sigma_o}{\sigma_c}$

FIG. 5



WOLLAK [2]
BALMER [4]
RICHART, ET AL. [3]
FREUDENTHAL [1]
δ
$\frac{\sigma_o}{\sigma_c}$
$-\frac{\tau_o}{\sigma_c}$
0 1 2 3 4 5 6 7 8 9 10 11
0 1 2 3 4 5 6 7

FIG. 6

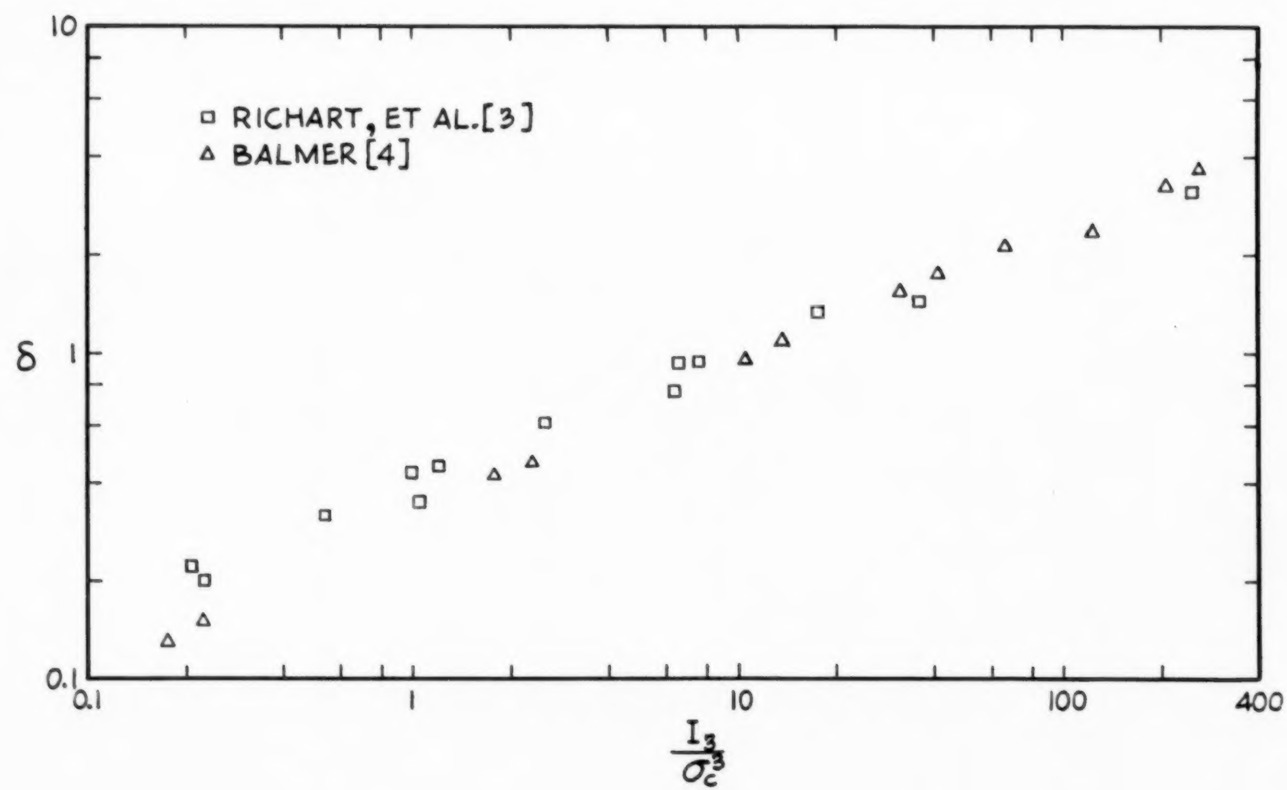


FIG. 7

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